Preprocessing Polynomials for Arithmetic Reasoning within the SMT-Solver STABLE

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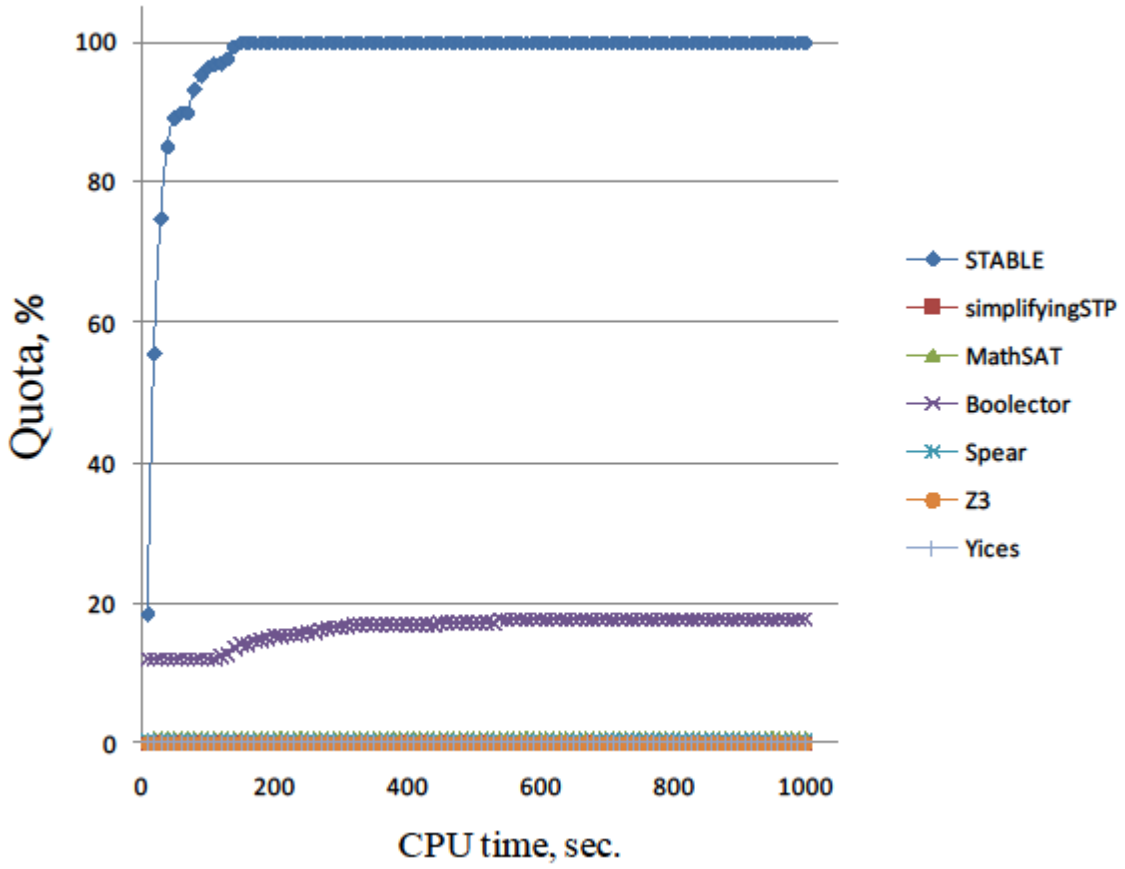
STABLE = SMT+ABL+GB



"STABLE: A new QF-BV SMT solver for hard verification problems combining Boolean reasoning with computer algebra", (DATE 2011), Grenoble, France.

Topic of this talk

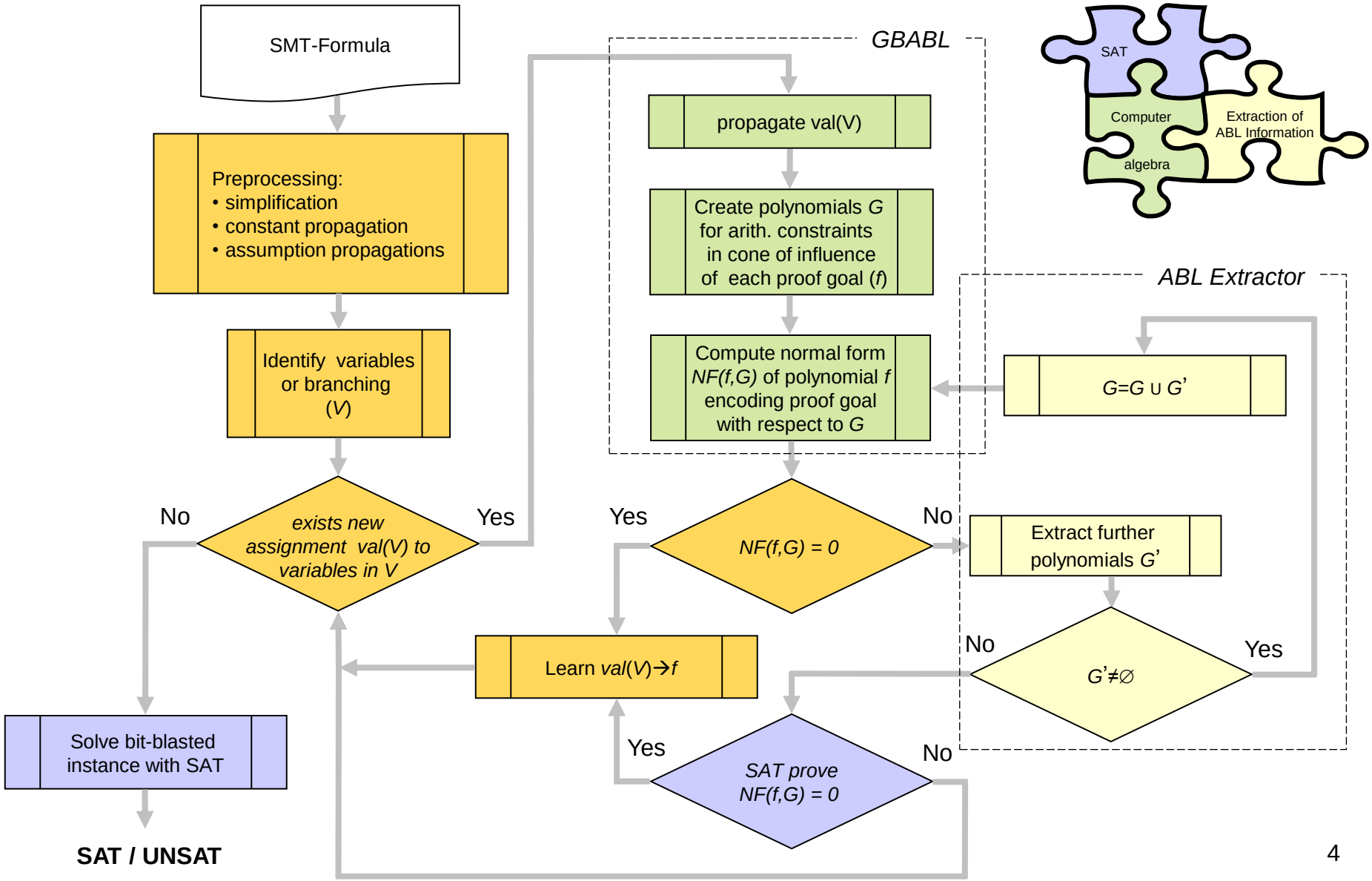
- The heart of STABLE is a computer-algebra-based engine. It incorporates an algebraic normal form algorithm that requires a polynomial model of the arithmetic parts of an SMT formula.

- The effectiveness of this algorithm may heavily depend on the initial formulation of the polynomials.

  - **Preprocessing of polynomials is crucial for successful performance of the normal form algorithm.**

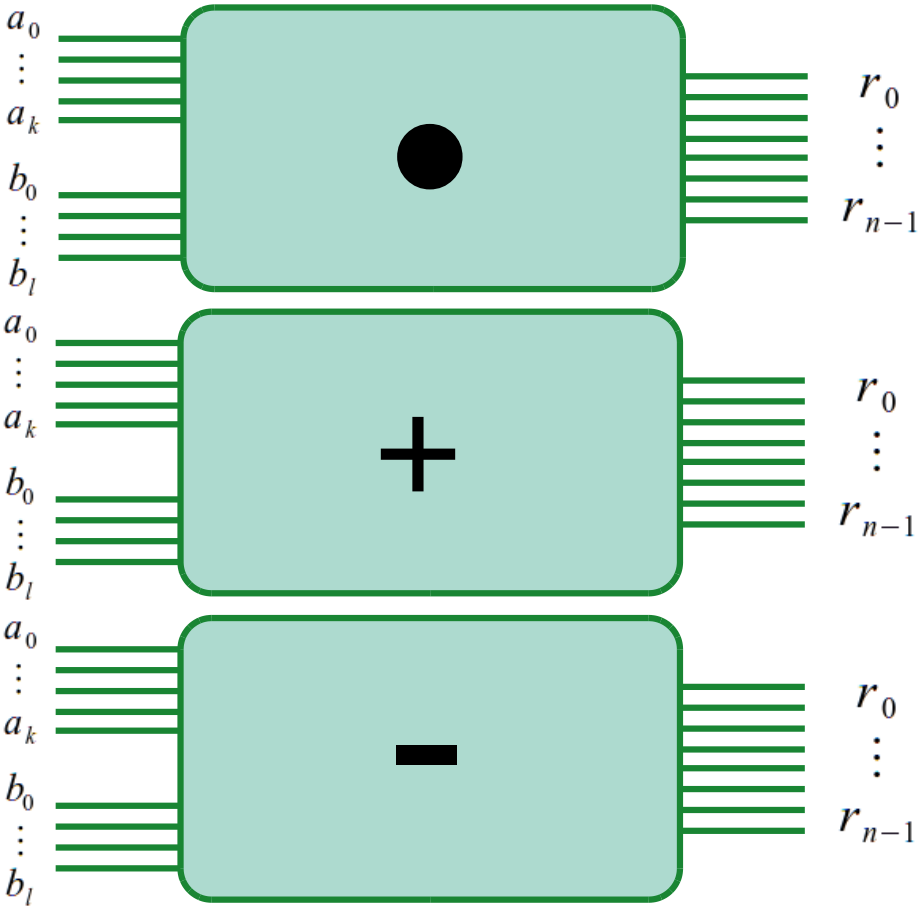
… however, for proper understanding of this problem, let us briefly recall how arithmetic models are treated in STABLE.

Design of the SMT-solver STABLE

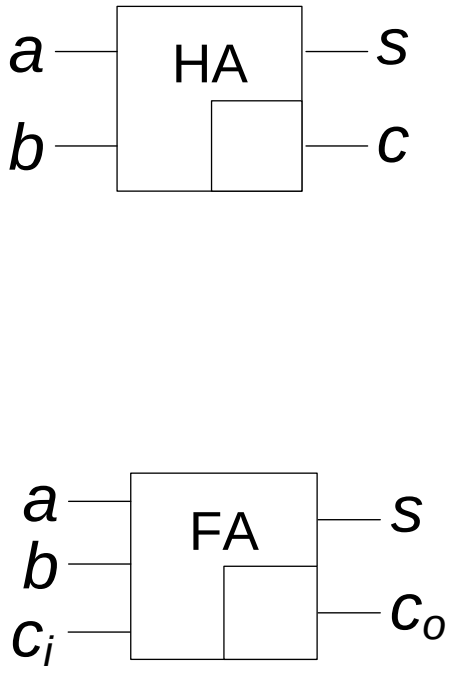


Modeling arithmetic problem parts

- Polynomial models available for:



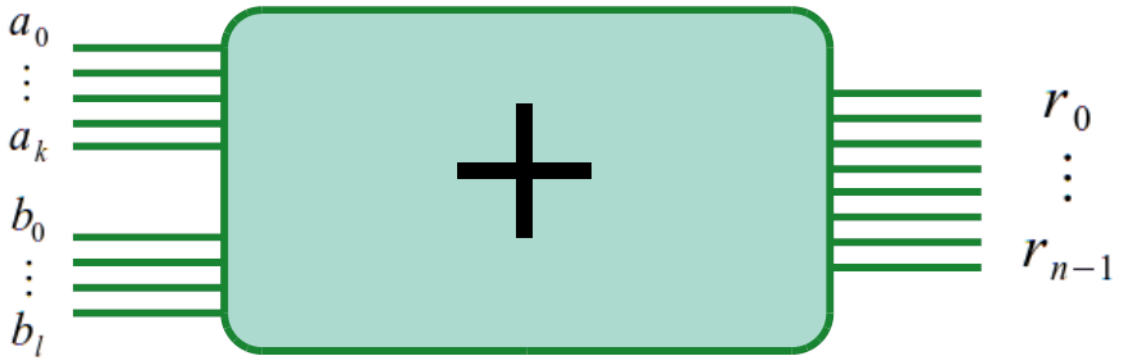
arithmetic word level



arithmetic bit level (ABL)

Modeling arithmetic problem parts

- Model each individual data path component



using polynomials over the ring $Z/2^N$

$$2^{n-1}r_{n-1}+\ldots+r_0-(2^k a_k+\ldots+a_0+2^l b_l+\ldots+b_0)+2^n s_n,\quad k<n,\quad l<n$$

$$2^{n-2}r_{n-2}+\ldots+r_0-(2^k a_k+\ldots+a_0+2^l b_l+\ldots+b_0)+2^{n-1} s_{n-1},\quad k<n-1,\quad l<n-1$$

$$\vdots$$

$$r_0-(a_0+b_0)+2s_1$$

Slack variables are artificial variables that are introduced to mimic the wrap around semantics of the components within a larger ring.

Application of Gröbner basis techniques

- Overall polynomial system

$$G = \left\{ g_{j,k} : \begin{array}{l} j = 1, \ldots, \text{number of components} \\ k = 1, \ldots, \text{bitwidth of } j\text{-th component} \end{array} \right\}$$

- Polynomial proof goal

$$g \in Z / 2^N [x_1, ..., x_n]$$

- Example: *k*-bit equality comparison

$$g = \sum_{i=0}^{k-1} 2^i (a_i - b_i)$$

Application of Gröbner basis techniques

- Validity of proof goal is equivalent to

$$V(\langle G \rangle) \subseteq V(g)$$

- Since G is a strong Gröbner basis this holds if and only if

$$NF(2^{N-n} g, G) = 0$$

- Model can be refined using bit-valued variables such that zero function test for $NF(2^{N-n} g, G)$ becomes unnecessary.

Application of Gröbner basis techniques

- Possible reasons for $NF(2^{N-n}g, G) \neq 0$:

  Outer: The arithmetic parts of a design and a property, out of which an SMT formula was generated, might include custom-designed components, non-arithmetic constraints, and bugs:

  - *Treated by polynomial extraction and resource-limited SAT checks.*

  Inner: Processing of slack variables is not powerful enough in polynomial systems of the computer-algebra engine:

  - *Explained on the next slides.*

Slack variables in STABLE

$$G_j^{(t)} : \sum_{i=0}^{t-1} 2^i r_i^{(j)} = f_j^{(t)}(a_1^{(j)}, a_2^{(j)}, \ldots, a_{m_j}^{(j)}) \bmod 2^t$$

$$\tilde{G}_j^{(t)} := \sum_{i=0}^{t-1} 2^i r_i^{(j)} - f_j^{(t)}(a_1^{(j)}, a_2^{(j)}, \ldots, a_{m_j}^{(j)}) + 2^t s_t^{(j)}$$

- Pro: Enables transformation of an arithmetic equation into an element of the polynomial ring over $Z / 2^N$.
- Contra: Slack variables are only visible inside the algebraic engine.
  Replacement of slack variables by their defining polynomials may eventually result in a computational blow-up.

➢ Generate as few slack variables as possible!

➢ Minimize the set of slack variables as much as possible!

Simplification of polynomials in STABLE

- **Estimate bounds**

  Compute an upper bound f_{ub} of $f(X)$ in the polynomial

$$p = \sum_{i=0}^{n-1} r_i - f(X) + 2^n s.$$

  The term $2^n s$ can be omitted if $f_{ub} < 2^n$.

  Example:

$$r_0 = x_1 \cdot x_2 \bmod 2 \quad \Rightarrow \quad r_0 - x_1 \cdot x_2 + 2s \in Z / 2^N [X]$$

$$x_1, x_2 \in \{0,1\} \quad \text{and} \quad (x_1 \cdot x_2)_{ub} < 2 \quad \Rightarrow \quad s = 0$$

$$r_0 - x_1 \cdot x_2$$

Simplification of polynomials in STABLE

- **Improve bounds**

  For a particular node, reuse the bounds computed in fan-ins and propagate them towards fan-outs.

  Example:

$$r_0 - (x_1 + x_2) + 2s \in Z / 2^N [X],$$

$$\text{where} \quad (x_1)_{ub} = 1 \quad \text{and} \quad (x_2)_{ub} = 0,$$

$$\text{then} \quad (x_1 + x_2)_{ub} = 1 \quad \text{and} \quad (x_1 + x_2)_{ub} < 2 \quad \Rightarrow \quad s = 0.$$

Simplification of polynomials in STABLE

- **Match polynomials**

  a) Detect common slack variables from different components.

  Example:

$$\begin{cases} p_1 = f_1(x) - 2^{n_1} s_1 \\ p_2 = f_2(x) - 2^{n_2} s_2 \end{cases}$$

$$\text{if} \quad f_1(x) = f_2(x) \quad \text{and} \quad n_1 = n_2 \quad \text{then} \quad s_1 = s_2 .$$

Simplification of polynomials in STABLE

- **Match polynomials**

  b) Reuse slack variables for different polynomials originating from the same SMT constraint.

Example:

$$r_0 + 2r_1 = (a_0 + b_0) + 2(a_1 + b_1) \bmod 2^2, \quad \text{with} \quad a_1 = 1, b_1 = 0$$

$$\begin{cases} p_1 = r_0 - (a_0 + b_0) + 2s_0 \\ p_2 = r_0 + 2r_1 - ((a_0 + b_0) + 2) + 4s_1 \end{cases}$$

However, if the partial assignment $(a_1 = 1, b_1 = 0)$ results in an overflow then it occurs simultaneously for both polynomials. Therefore, it always holds that $s_0 = s_1$.

Experimental results

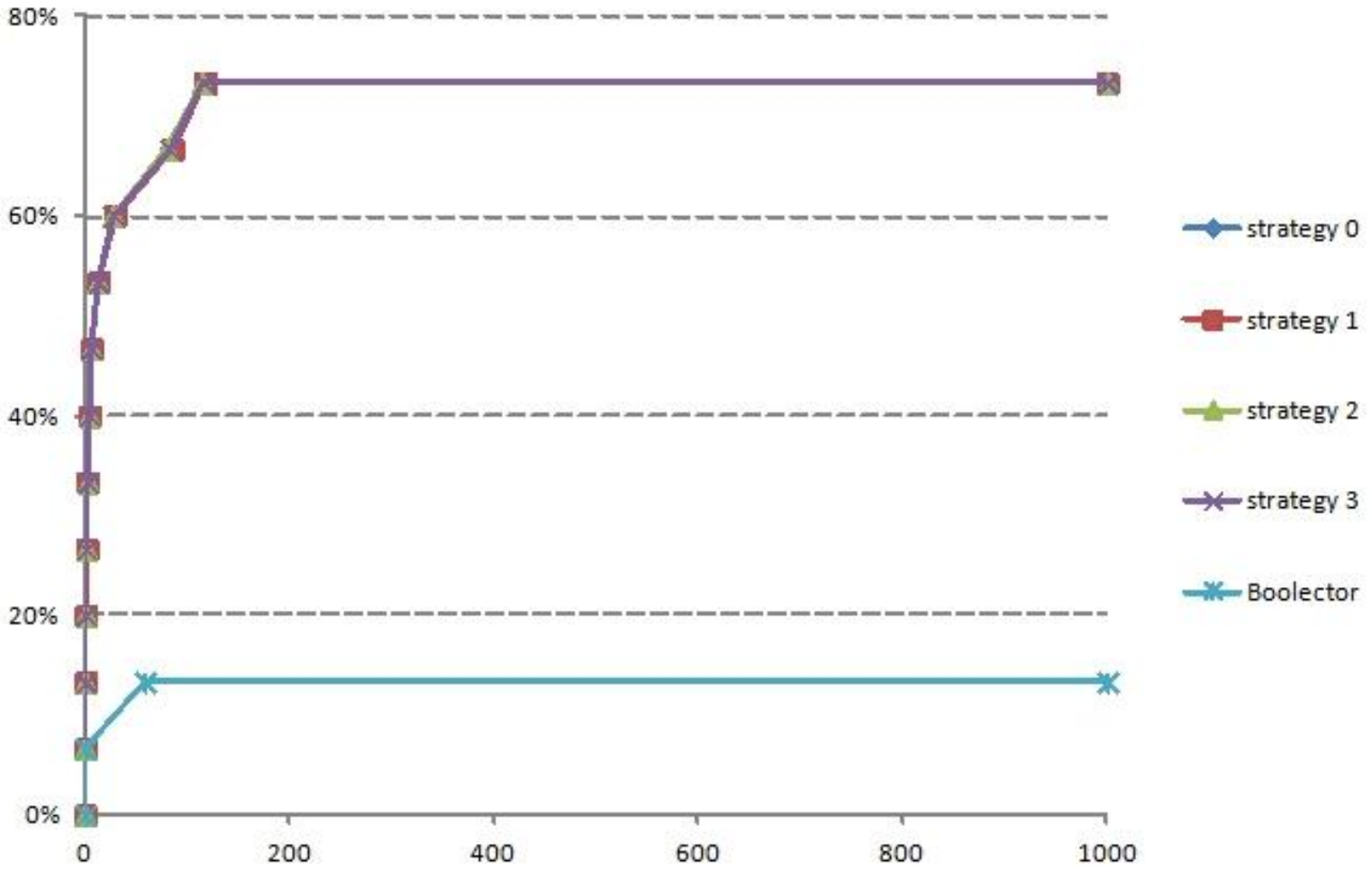
- The proposed simplification algorithms were categorized into four slack strategies in STABLE as follows:
  - *Strategy 0:* *no slack optimizations*
  - *Strategy 1:* *matching techniques*
  - *Strategy 2:* *bounding techniques*
  - *Strategy 3:* *matching and bounding techniques*

- Additionally, performance of STABLE was compared against Boolector (winner 2008 SMT-competition).

- All experiments were carried out on Intel Xeon CPU E5440 2,83 GHz 24 GB RAM running Linux.

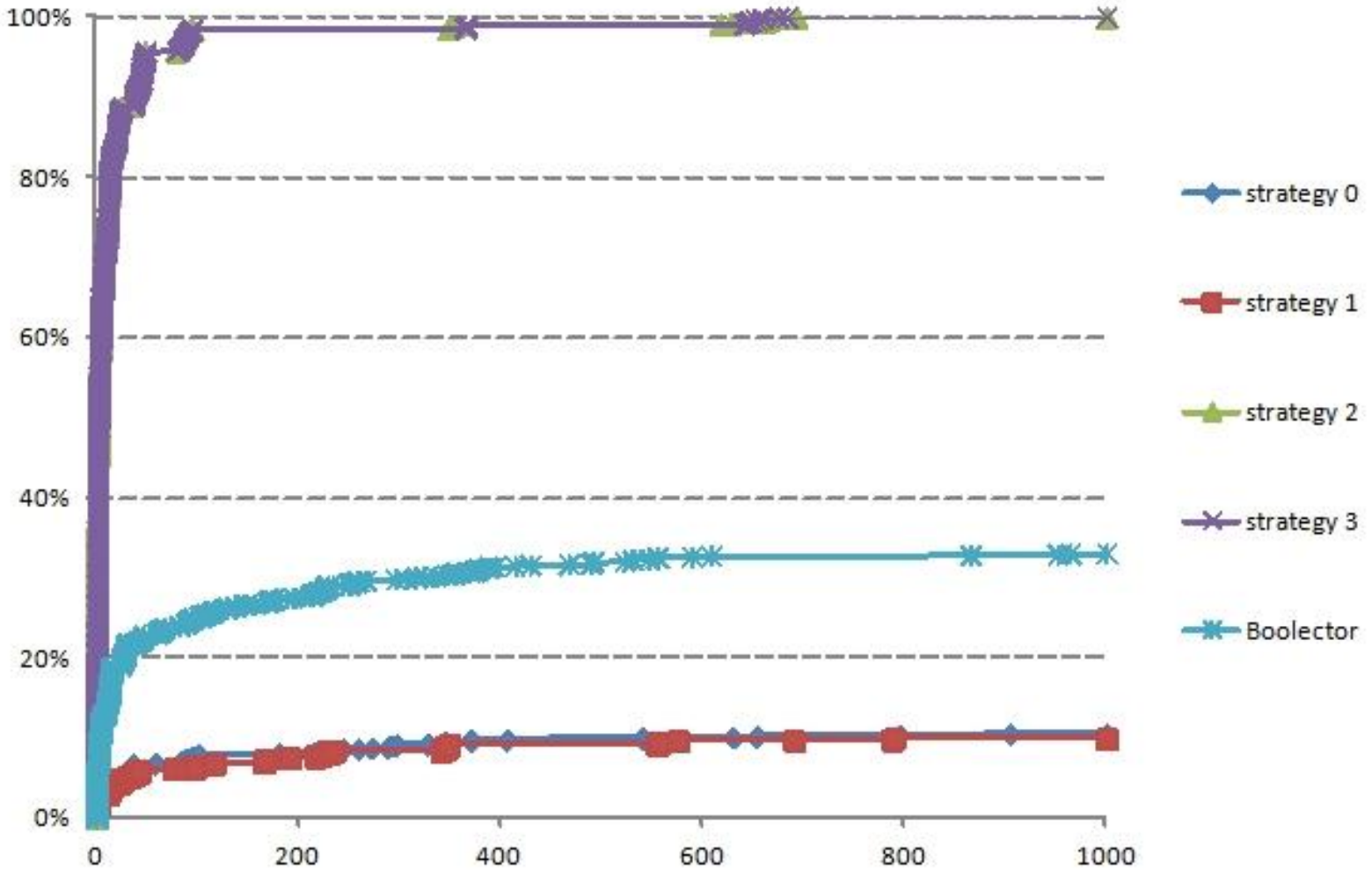
Experimental results

1) Suite of self-generated SMT instances that verify the function $f = a \cdot (b \cdot (c \cdot d))$ by a multiplier design with bit widths: 4 to 24.



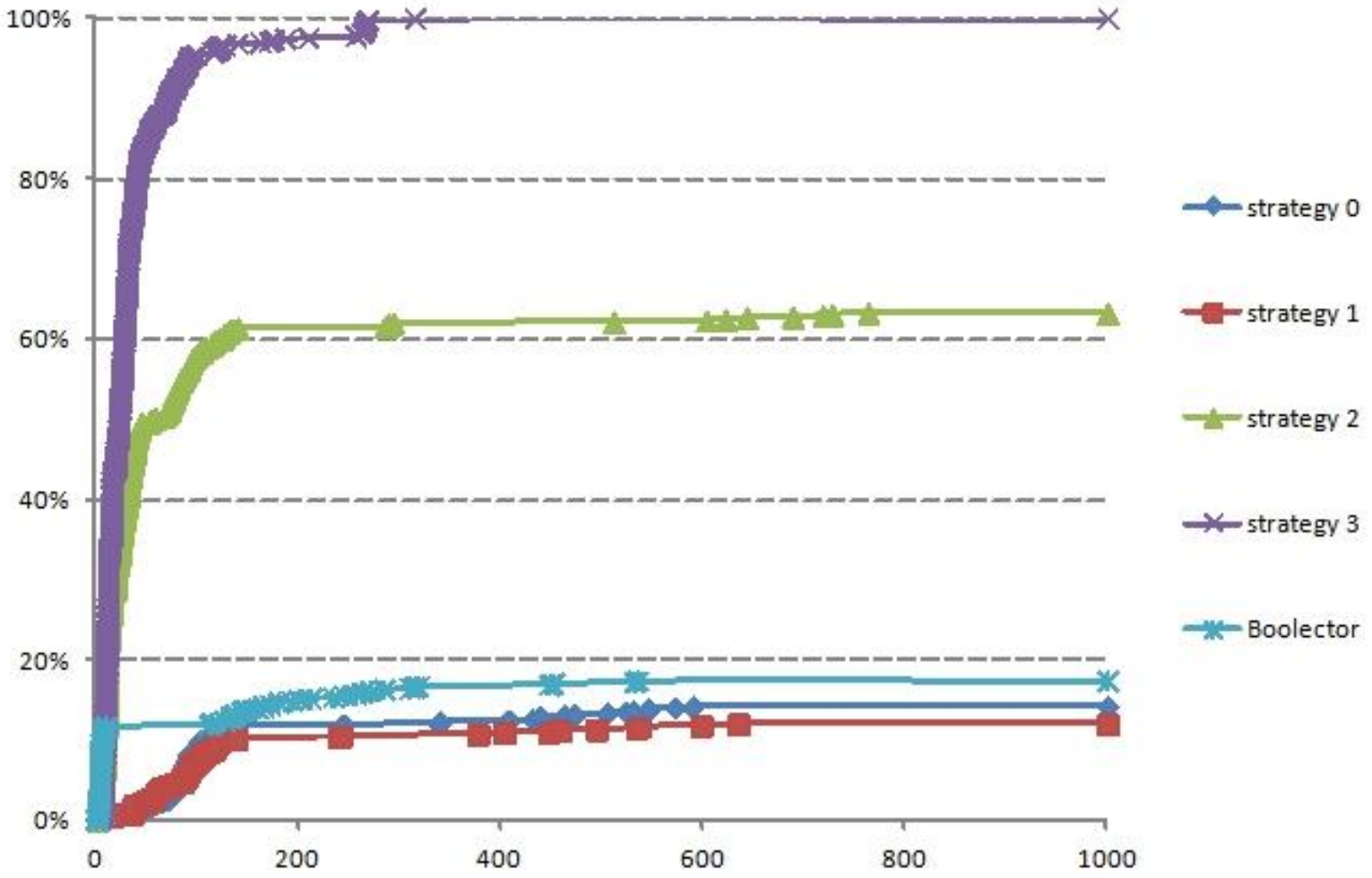
Experimental results

2) Suite of 1040 SMT instances that verify designs of industrial Booth-encoded multipliers for bit width from 4 up to 64 bits.



Experimental results

3) Suite of 640 SMT instances that verify arithmetic operations in an RTL design of the industrial TriCore processor.



Conclusion

- STABLE demonstrates outstanding performance when solving data-path verification problems at the register transfer level.

- Preprocessing techniques for polynomial models are vital for the practical efficiency of the overall SMT-based verification flow in STABLE.

Thank you for your attention!